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AFFINE TRANSFORMATIONS OF THE TANGENT BUNDLES WITH THE CONNECTION OF THE COMPLETE LIFT

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ABSTRACT

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GENERAL DESCRIPTION OF WORK

Historical overview and relevance of the research topic. The theory of tangent bundles over differentiable manifolds is one of the actively developing directions of the geometry of fiber spaces.

The tangent bundles of the first order of smooth manifolds arose in the middle of the 30s of the last century in connection with the creation of the theory of fiber spaces[1]. The tangent bundles of higher orders of T^kM were constructed by V.V. Wagner[2]. He called them holonomic compound varieties. The bundles that include the tangent bundles T^kM, called the bundle of p^v-velocities, were constructed by E. Ehresman[3],[4]. In 1953, A. Weyl introduced the bundles of A-close points, where A is the Weyl algebra[5]. The tangent bundles T^kM can be interpreted as bundles of "close points" A. Weyl. In 1974, A.P. Shirokov showed that on T^kM there exists a smooth structure over the algebra of pluriplane numbers $R(\varepsilon^k)$[6], and somewhat later A.P. Shirokov and V.V. Shurygin found structures of smooth manifolds on Weil fibrations[7],[8].

The differential geometry of tangent bundles began to develop from the late 1950s onwards after the publication of a series of works by Sh. Sasaki[9],[10]. In these papers, vertical, total lifts of vector and covector fields from the base *M* to the tangent bundle *TM* are introduced. Given a metric *g* on the basis of a Riemannian metric, Sasaki constructed a Riemannian metric on *TM*, which is called the Sasaki metric in the literature. K. Yano and S. Kobayashi[11],[12] introduced vertical, complete lifts of tensor fields and connections from the base *M* to the tangent bundle *TM*.

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[1]Steenrod, N. Topology of slanting works / N. Steenrod. - Moscow: Publishing House of Foreign Literature, 1953. - 274 p.

[2]Wagner, V.V. The theory of differential objects and the basis of differential geometry. Additions to the book Veblen O.and J. Whitehead, Foundations of Differential Geometry. Wagner. - Moscow: Izd-vo inostr. Lit.-ry, 1949. - P. 135-221.

[3]Ehresmann, C. Les prolongements d'une variete differentiable. I. Calcul des jets, prolongement principal / C. Ehresmann // C. E. Acad. Sci. - 1951. - No. 11. - P.598-600.

[4]Ehresmann C. Les prolongements d'une variete differentiable. II. L'espace des jets d'ordre r dedans Vm / C. Ehresmann // C. E. Acad. Sci. - 1951. - No. 15. - P.777 - 779.

[5]Weil, A. Theorie des points proches sur les varieties differentiables, Colloque internat. Center nat. Rech. Sci. / A. Weil // Geom. Different. - 1953. - Vol. 52- P. 111-117.

[6]Shirokov, A.P. A remark on structures in tangent bundles / A.P. Shirokov // Proceedings of the geom. Sem. M. VINITI AS SSR. - 1974. - T.5.- P. 311-318.

[7]Shirokov, A.P. Geometry of tangent bundles and spaces over algebras. Shirokov // Itogi Nauki i Tekhn. Ser. Probl. Geom. VINITI. - 1981. - №12.- With. 61-95.

[8]Shurygin, V.V. Varieties over local algebras equivalent to bundles of jets. Shurygin, "Izv. Universities. Mat. - 1992. - №10. - P.68-79.

[9]Sasaki, Sh. On the differential geometry of tangent bundles of Riemannian manifolds. I / Sh. Sasaki // Tohoku Math. J. - 1958. - No. 3. - P. 338-354.

[10]Sasaki, Sh. On the differential geometry of tangent bundles of Riemannian manifolds. II / Sh. Sasaki // Tohoku Math. J. - 1962. - Vol.14, No. 2. - P. 146-155.

[11]Yano, K. Prolongation of tensor fields and connective to tangent bundles. I. General theory / K. Yano, Sh. Kobayasi // J. Math. Soc. Japan. - 1966. - Vol. 18. - №2. - P. 194-201.

[12]Yano, K. Prolongation of tensor fields and connective to tangent bundles. II. Infinitesimal automorphisms / K. Yano, Sh. Kobayasi // J. Math. Soc. Japan. - 1966. - Vol. 18. - №3. - P. 236-246.

K. Yano and S. Ishihara[13] constructed horizontal lifts of tensor fields and connections in tangent bundles. A family of Riemannian metrics on *TM* that depend on three scalar fields, obtained from the metric defined on the basis of the bundle, was constructed by F.I. Kagan[14]. The Sasaki metric and the metric of a complete lift can be distinguished in this family as special cases for certain values of scalar fields.

In 1975, N.V. Talantova and A.P. Shirokov constructed a sinectic metric in the tangent bundle *TM*, starting from the Riemannian metric *g* and the symmetric tensor field of type (0,2) defined on the basis of *M*.

The results obtained in the study of tangent bundles of the first and second orders and the cotangent bundles are described in the monograph of K. Yano and S. Ishihara[15]. Constructs of lifts of tensor fields and linear connections from the base *M* to the tangent bundles T^kM and to the bundle of p^v-velocities are devoted to the works of A. Morimoto[16],[17]. He also constructed an extension of the linear connection defined on a smooth manifold *M* to the Weil bundle M^A[18]. Conditions are obtained for the connections that appear on M^A to be locally symmetric. The different structures on M^A were studied by V.V. Shurygin and his students[19], A.Ya. Sultanov[20], I. Kolarzh, P. Mihor, J. Slovak[21]. Vector and tensor bundles were studied by B.N. Shapukov and his students[22]. B.N. Shapukov built a complete lift of linear connection on tensor bundles. The semitangent structures on smooth manifolds were studied by V.V. Vishnevsky and his students[23],[24].

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[13]Yano, K. Horisontal lifts and connections to tangent bundles / K. Yano, S. Ishihara // J. Math. And Mech. - 1967. - Vol. 16, No. 9. - P. 1015-1029.

[14]Kagan, F.I. Riemannian metrics in the tangent bundle over a Riemannian manifold / F.I. Kagan // Izv. Universities. Mathematics. - 1973. - №6. - P.42-51.

[15]Yano, K. Tangent and cotangent bundles. Differential Geometry / K. Yano, S. Ishihara. - New York, Marcel Dekker, 1973. - 423 p.

[16]Morimoto, A. Prolongation of connections to tangent bundles of higher order / A. Morimoto // Nagoya Math. J. - 1970. - Vol. 40. - P. 99-120.

[17]Morimoto, A. Liftings of some type of tensor fields and connections to tangent bundles of pv-velocities / A. Morimoto // Nagoya Math. J. - 1970. - Vol. 40. - P. 13-31.

[18]Morimoto, A. Prolongation of connections to bundles of infinitely near points / A. Morimoto // J. Diff. Geom. - 1976. - Vol. 11. P. 479-498.

[19]Shurygin, V.V. Smooth varieties over local algebras and the Weil fibration. Shurygin // Itogi Nauki i Tekhn. Ser. Sovrem. mat. And its adj. Subject. Obz. VINITI. - 2002. - T. 73. - P. 162-236.

[20]Sultanov, A.Ya. Extensions of tensor fields and connections to Weil fibrations / A.Ya. Sultanov / / Izv. Universities. Mat. - 1999.- №9. - P. 64-72.

[21]Kolar, I. Natural operation in a differential geometry / I. Kolar, P. Michor, J. Slovak. - Berlin: Springer. - Verlag, 1993 - 437 p. 22Shapukov, B.N. Connections on differentiable fibrations. Shapukov // Itogi Nauki i Tekhn. Ser. Probl. Geom. - 1983. - T. 15. - C. 61-93.

[22]Shapukov, B.N. Lift connectivity on tensor bundles / B.N. Shapukov // Izv. Universities. Mat. - 1986. - № 12. - C. 70-72.

[23]Vishnevsky, V.V. Varieties over plural numbers and semitangent structures / VVVishnevsky // Itogi Nauki i Tekhniki. Problems of geometry. VINITI. - 1988. - T. 20 - P. 35-75.

[24]Vishnevsky, V.V. Spaces over algebras. Vishnevsky, A.P. Shirokov, V.V. Shurygin. - Kazan: Publishing House of Kazan University, 1984. - 264 p.

Along with the geometric objects on *TM* arising from objects defined on the basis of *M*, infinitesimal isometries, affine and projective transformations of these objects were studied.

The groups of motions in the Riemannian spaces were studied by B. Riemann, S. Lee, G. Fubini, I.P. Egorov and other geometers. Groups of projective, affine, conformal motions in pseudo-Riemannian spaces were studied in the papers of A.V. Aminova[25],[26],[27]. A major contribution to the theory of affine, projective motions was made by I.P. Egorov[28], G. Vrencanu, K. Yano and their students[29],[30].

One of the first works on the study of infinitesimal isometries in tangent bundles equipped with a Riemannian metric is the work of Sh. Sasaki[31]. K. Sato[32] obtained the general form of infinitesimal affine transformations in tangent bundles with the Sasaki metric. K. Yano and S. Kobayashi obtained a decomposition of infinitesimal affine transformations in *TM* with the connection of a complete lift that preserve the fibers. This problem was also solved in the general case[33]. They also investigated the projected infinitesimal isometries of the tangent bundle *TM* with the metric g^C of the total lift of the metric g defined on *M*. The complete solution of the problem of decomposing infinitesimal isometries in *TM* with the metric of the total lift g^C was given by Sh. Tanno[34].

A study of the infinitesimal projective, affine, generalized conformal, homothetic, isometric transformations in the tangent bundle *TM* equipped with the metric of a complete lift and the Sasaki metric is devoted to the papers of V.G. Podolsky[35].

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[25]Aminova, A.B. Groups of projective and affine motions in spaces of general theory of relativity, I / A.V. Aminova // Proceedings of the geometric seminar. VINITI. - 1974. - №6. - P. 317-346.

[26]Aminova, A.B. Groups of almost projective motions of spaces with affine connection. Aminov // Izv. Universities. Mat. - 1979. - №4. - P. 71-75.

[27]Aminova, A.V. Projective transformations of pseudo-Riemannian manifolds. Aminova. - Moscow: Janus-K, 2003. - 619 p.

[28]Egorov, I.P. Motions in spaces of affine connection / I.P. Egorov. - Kazan: Publishing house of the Kazan state university, 1965. - P. 5-179.

[29]Egorov, I.P. Motions in generalized differential-geometric spaces. Egorov. // The results of science. Algebra. Topology. Geometry. VINITI. - 1967. - P. 375-428.

[30]Egorov, I.P. Automorphisms in generalized spaces. Egorov // Results of science and technology. Problems of geometry. VINITI. - 1980.- T. 10. - P. 147-191.

[31]Sasaki, Sh. On the differential geometry of tangent bundles of Riemannian manifolds. II / Sh. Sasaki // Tohoku Math. J. - 1962. - Vol.14, No. 2. - P. 146-155.

[32]Sato, K. Infinitesimal affine transformations of the tangent bundles with Sasaki metric / K. Sato // Tohoku Math. J.- 1974.- 26.- No. 3.- P. 353-361.

[33]Yano, K. Prolongation of thensor fields and connective to tangent bundles. II. Infinitesimal automorphisms / K. Yano, Sh. Kobayasi // J. Math. Soc. Japan. - 1966. - Vol. 18. - №3. - P. 236-246.

[34]Tanno, S. Infinitesimal isometries on the tangent bundles with complete lift metric / S. Tanno // Tensor, N.S. - Vol.28. - 1974. - P. 139-144.

[35]Podolsky, V.G. Infinitesimal transformations in the tangent bundle with the metric of the total lift and the Sasaki metric / V.G. The Podolsky // Izv. Universities. Mathematics. - 1976. - №9. - P. 128-132.

The canonical decomposition of arbitrary infinitesimal projective and affine transformations on the tangent bundle *TM*, equipped with a lift of a linear connection ∇ defined on the basis of a fibration, was obtained by F.I. Kagan[36].

K. Yamaguchi[37] considered infinitesimal projective and conformal transformations in *TM* with the metric of the total lift g^C over a connected Riemannian manifold (M, g). The canonical decomposition of infinitesimal affine transformations in the tangent bundle *TM* with a sinectic connection in the sense of A.P. Shirokov was received by H. Shadyev[38]. The synectic metrics and their infinitesimal isometries are studied in S.Ya. Nus[39].

The study of tangent bundles and their automorphisms, provided with different geometric structures, is devoted to a series of papers by VI. Panzhensky[40]. The motions in the tangent bundles TM with a special metric were studied by O.P. Surina[41]. The motions in the tangent bundles preserving the orthogonal and tangent structures were considered by R.Kh. Ibrahimova[42]. The structure of the Lie algebra of holomorphic vector fields on Weil fibrations is considered by A.Ya. Sultanov[43]. N.D. Nikitin[44] studied infinitesimal affine transformations of non-linear connection on TM. Infinitezimous automorphisms of an almost symplectic structure on the tangent bundle of a generalized Lagrangian space were considered by M.V. Sorokina[45].

[36]Kagan, F.I. Canonical decomposition of projectively-Killing and affine-Killing vectors on the tangent bundle / F.I. Kagan // Matem. notes. - 1976. - 19: 2.- P. 247-258.

[37]Yamauchi, K. Infinitesimal projective and conformal transformations in a tangent bundle / K. Yamauchi // Sci. Rep. Kagoshima Univ. - 1983. - № 32. - P. 47-58.

[38]Shadyev, H. Affine collineation of a synectic connection in the tangent bundle / X. Shadyev // Tr. - 1984. - T. 16. - P. 117-127.

[39]Nus, S.Ya. Special Riemannian metrics in tangent bundles: Dis kh. To the soot. Scientist. step. Candidate of physico-mathematical sciences: 01.01.04 / Svetlana Yakovlevna Nus; Kazan State University. Un-t. - Kazan, 1985. - 95 liters.

[40]Panzhensky, V.I. Infinitesimal automorphisms of metric spaces of Finsler type Panzhensky // Results of science and technology. Ser. Sovrem. Mathematics and its applications. Subject. Reviews. - 2009. - T. 123. - C. 81-109.

[41]Surina, O.P. On motions in spaces with the metric gij (x, y) = e^{2v} (x, y) γ_{ij} (x) / OP. Surina // Geometry of generalized spaces. Interuniversity sb. Scientific works Penza. State. Ped. In-t. - 1992. - P. 96-100.

[42]Ibragimova, R.Kh. Motions on tangent bundles that preserve the orthogonal and tangent structures of / R.X. Ibrahimov, "Izv. Universities. Mathematics. - 1996. - № 8. - P. 29-34.

[43]Sultanov, A.Ya. Holomorphic affine vector fields on Weil bundles / A.Ya. Sultanov // Matem. notes. - 2012. - 91: 6. - P. 896-899.

[44]Nikitin, N.D. Infinitesimal Motions in Spaces of Nonlinear Connectedness. Nikitin // Motions in generalized spaces. Interuniversity sb. Scientific works Penza. State. Ped. University. - 1999. - P. 93-101.

[45]Sorokina, M.V. On infinitesimal automorphisms of an almost symplectic structure on the tangent bundle of a generalized Lagrangian space. Sorokin // Sci. App. Kazan. State. University. Ser. Phys.-Math. science. - 2005. - 147: 1. - P. 154-158.

O.A. Monakhova studied infinitesimal affine transformations of the bundle of doubly covariant tensors with a connection horizontal lift[46]. Similar studies were carried out in the works of N.A. Osminina[47], N.I. Manina and A.Ya. Sultanov[48], K.M. Budanov and A.Ya. Sultanov[49].

Proceeding from the above, it can be concluded that the theme of this dissertation is relevant.

The thesis on its topic refers to the theory of tangent bundles of differentiable manifolds.

The aim of the thesis is to study the dimensions of groups of affine transformations in tangent bundles with the connection of a complete lift.

The main tasks of the thesis work.

1. Investigation of the integrability conditions for the equations of infinitesimal affine transformations of tangent bundles with a connection of the complete lift (TM, $\nabla^{(0)}$).

2. A study of the structure of Lie algebras of infinitesimal affine transformations of tangent bundles with a connection of a complete lift.

3. Establishment of maximal dimensions of Lie algebras of infinitesimal affine transformations of spaces (TM, $\nabla^{(0)}$) and the accuracy of these dimensions.

4. The study of gaps in the distribution of groups of affine transformations in the tangent bundle TM with the connection of the complete lift $\nabla^{(0)}$.

Methodology and methods of research. The thesis uses methods of local differential geometry, the apparatus of tensor analysis and the Lie derivative is used. Functions, tensor fields are assumed to be smooth of class C , and linear connections with a zero tensor torsion field.

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[46]Monakhova, O.A. Infinitesimal affine transformations of the bundle of doubly covariant tensors with a connection of a horizontal lift: dis. To the soot. Scientist. step. Candidate of physico-mathematical sciences: 01.01.04 / Oksana Aleksandrovna Monakhova; Kazan State University. Un-t. - Kazan, 2004. - 105 liters.

[47]Osminina, N.A. Infinitesimal affine transformations of a tangent bundle of second order with a synectic connection: Dis kand. To the soot. Scientist. step. Candidate of physico-mathematical sciences: 01.01.04 / Natalia Aleksandrovna Osminina; Kazan State University. Un-t. - - Kazan, 2003. - 103 liters.

[48]Manina, NI, Infinitesimal affine transformations of a tangent bundle of second order with a connection of a horizontal lift / NI. Manina, A.Ya. Sultanov / / Izv. Universities. Mat. - 2011. - №9. - P. 62-69.

[49]Budanov, K.M. Infinitesimal affine transformations of the second order Weyl bundle with a complete lift connection / KM Budanov, A.Ya. Sultanov // Izv. Universities. Mat. - 2015. - № 12. - P. 3-13.

Scientific novelty. In the thesis the following results were obtained:

1. We introduce $V\gamma^r$- and $H\gamma^r$-lifts of tensor fields of the type $(1, r)$ ($r > 1$), which are generalizations of $V\gamma$- and $H\gamma$-lifts of tensor fields of the type (1,1), and some of their properties are proved.
2. Sharp upper bounds for the dimensions of Lie algebras of arbitrary infinitesimal affine transformations are obtained and the ideals of the Lie algebras of the projected infinitesimal affine transformations of the tangent bundle TM with the connection of the complete lift $\nabla^{(0)}$ are distinguished.
3. A gap is established in the distribution of the maximal dimensions of groups of affine transformations in tangent bundles with the connection of a complete lift.
4. We study groups of affine transformations over two-dimensional nonplanar maximally movable spaces.

Theoretical and practical significance of the work. Dissertational work is of a theoretical nature. Its results can be used to further develop the theory of fiber spaces with connections and metrics, and also in the learning process when reading special courses and elective courses for mathematic students.

Approbation of work. The results of the dissertation work were reported at the international conference of students and graduate students "Mathematics and its applications in modern science and practice" (Kursk, YuSGU, 2013), at the international scientific and technical conference "Analytical and numerical methods for modeling natural-science and social problems" (Penza, PSU , 2013), at the international conference "Lomonosov Readings in Altai: Fundamental Problems of Science and Education" (Barnaul, AlSU, 2014, 2015), at the International Conference on Algebra, Analysis and Geometry on the Jubilee Masters of the Kazan University, mathematicians Peter Alekseevich (1895-1944) and Alexander Petrovich (1926-1998) Shirokovs, and the youth school-conference on algebra, analysis, geometry (Kazan, KFU, 2016), at the geometrical seminar of the Department of Geometry and Mathematical Analysis of CCGT (headed by Prof. VI Panzhensky and Prof. A.Ya. Sultanov).

The main results of the thesis are reflected in 14 published works of the author, the list of which is given at the end of the dissertation. Of these, six articles are published in journals included in the list of VAK.

Structure and amount of work. The thesis is presented on page 131 and consists of an introduction, three chapters containing 18 paragraphs, a bibliographic list and a list of the author's publications on the research topic. The bibliography contains 66 titles of works of domestic and foreign authors.

Abstract summary of the dissertation

The introduction gives an overview of the results of the problem studied and summarizes the main results of the thesis.

The first chapter of the thesis is devoted to the study of extensions of tensor fields from a smooth manifold to its tangent bundle. §§1.1-1.4 are of a referential nature.

In §1.1 we define the tangent bundle TM over a differentiable manifold M, we introduce the concepts of vertical and complete lifts of functions from a base to a tangent bundle.

In §1.2 we describe a smooth structure on the tangent bundle over the algebra of dual numbers.

In §1.3 vertical and complete lifts of vector and covector fields on *TM* are introduced and some of their properties are proved, and also definitions of the natural lifts of tensor fields of type $(0, s)$ and $(1, s)$ $(s \geq 1)$ on *TM* generated by elements of the algebra of dual numbers and the vector space of linear forms defined on the algebra of dual numbers and taking values in the field *R* of real numbers.

In §1.4 we define a complete lift of a linear connection c of a manifold *M* into the tangent bundle *TM*, local connectivity components and components of the tensor fields of curvature and torsion in the natural movable frame are given. A linear connection is also called an affine connection. It should be noted that sometimes in the scientific literature the concepts of affine and linear connections are distinguished. We will not do this.

§1.5 is devoted to the study of vertical-vector lifts of tensor fields of type (1,1) with of the base into the tangent bundle. The commutators of $V\gamma$-lifts, vertical and total lifts of vector fields on *TM*, given in § 1.3, necessary for the study of infinitesimal affine transformations, are calculated, and some of their properties are proved.

In §1.6 we introduce a generalization of $V\gamma$-lifts - $V\gamma^r$-lifts of tensor fields of type $(1, r)$ $(r > 1)$ on the tangent bundle and prove their properties.

In §1.7 we consider horizontal vector lifts of tensor fields of type (1,1), and in §1.8 we define the $H\gamma^r$-lift of tensor fields of type $(1, r)$ $(r > 1)$ with *M* in TM, which is a generalization of the Hγ-lift of the tensor field type (1,1), introduced by Sh. Tanno in 1974. For these lifts, the Lie derivatives and covariant derivatives with respect to vertical and complete lifts of vector fields on *TM* are computed.

In the second chapter we study infinitesimal affine transformations of tangent bundles with a connection of a complete lift over a non-projective curve-base.

In §§2.1-2.2 we consider the integrability conditions for the equations of infinitesimal affine transformations, taking into account the structure of the canonical decomposition of an arbitrary infinitesimal affine transformation of the space *TM* equipped with a connection of the complete lift $\nabla^{(0)}$. It is proved that the canonical decomposition is unique. Necessary and sufficient conditions imposed on the tensor fields participating in this expansion are obtained.

In §2.3 we investigate the structure of the Lie algebra $\tilde{L}$ of infinitesimal affine transformations of tangent bundles with a connection of a complete lift.

In §§2.4-2.5 sharp estimates are obtained from above of the dimensions of the Lie algebras $\tilde{L}$ of infinitesimal affine transformations of the spaces $(TM, \nabla^{(0)})$ with a non-projective Euclidean base. Proven

THEOREM 2.4.2. The maximal dimension of the Lie algebra $\tilde{L}$ of infinitesimal affine transformations of the tangent bundle *TM* with the connection of the total lift $\nabla^{(0)}$ over the non-objective Euclidean Schouten-Stroyk space of the first type is exactly $4n^2 - 9n + 14$ $(n > 2)$, where n is the dimension of the base *M*.

THEOREM 2.5.1. The maximal dimension of the Lie algebra $\tilde{L}$ of infinitesimal affine transformations of the tangent bundle *TM* with the connection of the total lift $\nabla^{(0)}$ over the non-projective Euclidean Schouten-Stroyk base of the second type is at most $4n^2 - 13n + 22$ ($n > 3$), where n is the dimension of the base M.

In the third chapter, we study the groups of affine transformations and their Lie algebras of infinitesimal affine transformations of tangent bundles TM equipped with complete lifts $\nabla^{(0)}$ of torsion-free linear connections defined on the basis of M, provided that the connection ∇ is projectively Euclidean.

In §3.1 the maximal dimension of the Lie algebras $\tilde{L}$ of infinitesimal affine transformations is established in the case when the base M is an equiprojective space. Namely, it is proved

THEOREM 3.1.3. The maximal dimension of the Lie algebras of the infinitesimal affine transformations of the space (TM, $\nabla^{(0)}$) over the projectively Euclidean base (M, ∇) whose Ricci tensor field is symmetric is exactly $2n^2 + 1$ ($n > 2$), where n is the dimension of the base M .

In §3.2 we investigate the question of establishing the upper bound of the dimension of the Lie algebra $\tilde{L}$ of infinitesimal affine transformations in (TM, $\nabla^{(0)}$) with a projective Euclidean base with asymmetric Ricci tensor field. The following statements are proved:

THEOREM 3.2.1. If the base of the bundle (M, ∇) is a projective Euclidean space, then any infinitesimal affine transformation $\tilde{X}$ of the tangent bundle TM with the connection of the total lift ∇ (0) has the form:

$$\tilde{X} = X^{(0)} + Y^{(1)} + G^{V\gamma},$$

where $X, Y \in \beth^1_0(M)$, $G \in \beth^1_1(M)$.

THEOREM 3.2.3. Let ∇ be a linear connection defined on the base M of the bundle TM such that $\mathrm{rangRic}^+ = 1$, $\mathrm{rangRic}^- = 2$, and there exist vector fields X, Y defined in some neighborhood U of $p \in M$ that form a linearly independent system, and $[X, Y] = 0$ satisfying the following condition:

$\mathrm{Ric}^+(X, X) \neq 0$, $\mathrm{Ric}^-(X, Y) \neq 0$.

Then the maximal dimension of the Lie algebras $\tilde{L}$ of infinitesimal affine transformations of the space (TM, $\nabla^{(0)}$) is exactly $2n^2 - 2n + 3$ ($n > 2$), where n is the dimension of the base M.

THEOREM 3.2.4. Let ∇ be a linear connection defined on the base M of the bundle TM such that $\mathrm{rangRic}^+ = 1$, $\mathrm{rangRic}^- = 2$ and for any vector fields X, Y the following condition holds:

If $\mathrm{Ric}^+(X, X) \neq 0$, then $\mathrm{Ric}^-(X, Y) = 0$.

Then the dimension of the Lie algebras $\tilde{L}$ of infinitesimal affine transformations of the spaces (TM, $\nabla^{(0)}$) does not exceed $2n^2-4n + 7$ ($n > 2$), where n is the dimension of the base M.

As a consequence of these theorems, we have proved

THEOREM 3.2.5. The maximal dimension of the Lie algebras of infinitesimal affine transformations of the tangent bundle TM with the connection of the total lift $\nabla^{(0)}$ over the projective Euclidean base with the asymmetric tensor Ricci field is equal to $2n^2-2n + 3$ ($n > 2$).

In §3.3 we describe the projected affine vector fields and ideals of the Lie algebra of projected infinitesimal affine transformations of tangent bundles with a connection of the total lift. Proven

Proposition 3.3.2. In the Lie algebra K_1 of all projected infinitesimal affine transformations of the form $X^{(0)} + Y^{(1)} + G^{V\gamma}$, where $X, Y \in \beth_0^1(M)$, $G \in \beth_1^1(M)$ of the tangent bundle TM with the connection $\nabla^{(0)}$ single out a chain of nested ideals

$$K_1 \supset K_{11} = \{Y^{(1)} + G^{V\gamma}\} \supset L^1 = \{Y^{(1)}\}.$$

The ideal L^1 is also an ideal of the algebra K_1.

It is shown that in the general case the last ideal L^1 in this chain of embedded ideals may not be an ideal of the original algebra K_1.

§3.4 is devoted to the establishment of the maximal dimension of Lie algebras $\tilde{L}$ of infinitesimal affine transformations and to the study of lacunae in the distribution of groups of affine transformations of spaces $(TM, \nabla^{(0)})$. The following statements are proved:

Theorem 3.4.1. The maximal dimension of the Lie algebras of infinitesimal affine transformations in tangent bundles TM with the connection of the total lift $\nabla^{(0)}$ with the non-zero tensor curvature field is exactly $4n^2 - 9n + 14$ ($n > 2$).

THEOREM 3.4.2. The maximal dimension of the affine transformation groups in the tangent bundles TM with the connection of the total lift $\nabla^{(0)}$ of the connection ∇ to the non-zero tensor curvature field is exactly $4n^2 - 9n + 14$ ($n > 2$).

Theorem 3.4.3. There is no tangent bundle TM with a connection of the total lift $\nabla^{(0)}$, where ∇ is a projectively Euclidean linear connection whose affine transformation group has maximal dimension r satisfying the inequalities

$$2n^2 - 2n + 3 < r < 2n^2 - 1 \ (n > 2).$$

THEOREM 3.4.4. There is no tangent bundle TM with the connection of the total lift $\nabla^{(0)}$ whose affine transformations have the maximal dimension r satisfying the inequalities

$$4n^2 - 9n + 14 < r < 2n\,(2n + 1) \ (n > 2).$$

In §3.5 we study the maximal dimensions of the groups of affine transformations of the spaces $(TM_2, \nabla^{(0)})$ when the base M is a two-dimensional maximally movable space of linear connection.

Proven

THEOREM 3.5.4. The Lie algebra of all infinitesimal affine transformations of the tangent bundle TM_2 equipped with a complete lift of a maximally movable linear connection with a non-zero tensor curvature field has dimension 9.

THEOREM 3.5.5. The group of affine transformations of the tangent bundle TM_2 equipped with a complete lift of maximally movable linear connection with a non-zero tensor curvature field has dimension 9.

I.P. Egorov established three types of two-dimensional maximally-mobile spaces of linear connection. In the dissertation the question of the solvability of Lie algebras of infinitesimal affine transformations over these spaces is investigated. Proven

THEOREM 3.5.6. The Lie algebras $\tilde{L}$ of infinitesimal affine transformations of the spaces $(TM_2, \nabla^{(0)})$ over two-dimensional maximally movable spaces (M_2, ∇) are solvable.

PUBLICATIONS OF THE AUTHOR BY THE THEME OF DISSERTATION

Works published in the leading peer-reviewed scientific journals and publications (in accordance with the List of the HAC)

1. Sultanova, G.A. On some properties of lifts of tensor fields in tangent bundles. Sultanova // Bulletin of the Pedagogical University. - Dushanbe, 2013. - No. 5 (54). - P. 87-89.

2. Sultanova, G.A. Some lifts of tensor fields of type (1, *r*) c of the base into its tangent bundle. Sultanova // Proceedings of higher educational institutions. The Volga region. Phys.-Math. science. - 2014. - No. 1 (29). - P. 54-63.

3. Sultanov, A.Ya. Estimate of the dimensions of the Lie algebra of infinitesimal affine transformations of the tangent bundle *T(M)* with the connection of the total lift / A.Ya. Sultanov, G.A. Sultanova // Scientific notes of Kazan University. Ser. Phys.-Math. science. - 2014. - Volume 156. - Book 2. - P. 43-54.

4. Sultanova, G.A. On horizontal-vector liftings of tensor fields from a base to its tangent bundle / G.A. Sultanova // Bulletin of the BFU im. I.Kant. Ser. Phys.-Math. science. - 2015. - Issue. №10. - P. 80-89.

5. Sultanova, G.A. On the dimensions of Lie algebras of automorphisms in tangent bundles with a connection of a complete lift over a projectively Euclidean base / G.A. Sultanova // Dalnevost. Math. Journal. - 2016. - T.16. - №1. - P. 83-95.

6. Sultanova, G.A. On Lie algebras of infinitesimal affine transformations in tangent bundles with a connection of a complete lift. Sultanova // Proceedings of higher educational institutions. The Volga region. Phys.-Math. science. - 2016. - No.4 (40). - P. 38-50.

Works published in other editions

7. Sultanova, G.A. Integrability conditions for the equations of motion in a tangent bundle with a connection of a complete lift. Sultanova // Materials of the II International Conference of Students and PhD students "Mathematics and its applications in modern science and practice". - Kursk, 2013. - P. 83-86.

8. Sultanova, G.A. Modeling of tangent bundles by Weil fibrations / G.A. Sultanova // VIII International Scientific and Technical Conference "Analytical and Numerical Methods for Modeling Natural and Social Problems" (ACM-2013). - Penza, 2013. - P. 72-78.

9. Sultanova, G.A. Properties of $H\gamma$ -lifts of tensor fields of type (1,1) in tangent bundles. Sultanova // Issues of Education and Science: Theoretical and Methodical Aspects: Sat. Sci. Tr. On the materials of the Intern. Scientific-practical. Conf. May 31, 2014 - Tambov, 2014. - Part 9. - P. 139-142.

10. Sultanova, G.A. On some subalgebras of the Lie algebra of infinitesimal affine transformations of the tangent bundle *TM* with the connection of the total lift Sultanova // Collection of scientific articles of the international conference "Lomonosov Readings in Altai: Fundamental Problems of Science and Education". Section "Analysis, geometry and topology". - Barnaul: Publishing house Alt. Univ., 2014. - P. 378-381.

11. Sultanova, G.A. On some properties of $H\gamma^r$-lifts of tensor fields of type (1, *r*), (r> 1) from the base to its tangent bundle / G.A. Sultanova // Collection of scientific articles of the international conference "Lomonosov Readings in Altai: Fundamental Problems of Science and

Education". Section "Analysis, geometry and topology". - Barnaul: Publishing house Alt. Un-ta, 2015. - P. 563-566.

12. Sultanova, G.A. On groups of motions in tangent bundles with a connection of a complete lift over two-dimensional maximally mobile affinely connected spaces. Sultanova // Diff. Geom. Manifolds of figures. - 2015. - Issue 46. - P. 153-161.

13. Sultanova, G.A. Motions in tangent bundles with connectedness of a complete lift over a projective-Euclidean base / G.A. Sultanova // Materials of Int. Conf. On algebra, analysis and geometry, dedicated to the jubilees of prof. Kazan University of Mathematics PA. And A.P. Shirokov, and the youth school-conference on algebra, analysis, geometry. - 2016. - P. 322-323.

14. Sultanova, G.A. On the estimation of the dimensions of Lie algebras of infinitesimal automorphisms of tangent bundles with a connection of a complete lift over a non-projective Euclidean base / G.A. Sultanova // Diff. Geom. Manifolds of figures. - 2016. - Issue 47. - P. 146-153.